On ideal-function-like functions

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(Abstract)

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Let k be an algebraic number field of degree m over Q, let K be an arbitrary finite extension of k of degree n, and let K^* be the smallest Galois extension of k belonging to k (the compositum of all conjugates of K/k). Let $\mathfrak{G} = \mathrm{Gal}(K^*/k)$ be the corresponding Galois group which we view as embedded in the n-th symmetric group as a transitive subgroup. Let $\mathfrak{G} = \sum_{j=1}^{h} \mathfrak{C}_j$

be the decomposition of $\mathfrak{G}$ into conjugate classes, $\mathfrak{C}_1 = 1$. We divide all prime ideals other than those (which we denote by $\mathfrak{P}_0$) dividing the relative discriminant $D_{K/k}$ into h classe whose representatives we denote by $\mathfrak{P}_j$. Now define the ideal-function-like function g (in conformity with [B-R], incorporating [S1], [S2], [H-S]) by

$$g(\mathfrak{P}_j) = \lambda_j^{-1} > 0, \; j = 0,1,\ldots,h$$

at primes, and then define multiplicatively for all ideals. The two-fold aim of this paper is to study the rather complicated main term as well as the order of the error term of the asymptoti formula for the sums

$$A_1(x) = \sum_{N\mathfrak{a}\cdot g(\mathfrak{a}) \le x} 1, \qquad B_1(x) = \sum_{N\mathfrak{a} \le x} g(\mathfrak{a}).$$

This covers, in particular, the generalized divisor function $T^c(\mathfrak{a})$, $c>0$, where $T(\mathfrak{a}) = T_\kappa(\mathfrak{a})$ denotes the number of representations of the integral ideal $\mathfrak{a}$ of k as the product of κ integral ideals of K ([H-S], [S1], [S2]). The proof goes on the similar lines as those in [S1], [S2] plus [B-R], and uses [H].

In a similar setting (with P a subfield of k) one can consider the sums as in [S1]:

$$A_2(x) = \sum_{N\mathfrak{n} \cdot F(\mathfrak{n})^c \le x} 1, \quad B_2(x) = \sum_{N\mathfrak{n}\le x} F(\mathfrak{n})^c ,$$

where

$$F(\mathfrak{n}) = \sum_{N_{k/P}\mathfrak{n} = \mathfrak{a}} g(\mathfrak{a}).$$

Details will appear elsewhere.

References

[B-R] R. Balasubramanian and K. Ramachandra, On the number of integers n such that $nd(n)\le x$, Acta Arith. 49 (1988), 313-322.

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[H-S] H. Hasse and Z. Suetuna, Ein allgemeines Teilerproblem der Idealtheorie, J. Fac. Sci. Univ. Tokyo 2 (1931), 133-154.

[S1] Z. Suetuna, Ueber die Anzahl der Idealteiler, ibid. 2 (1931), 155-177.

[S2] Z. Suetuna, Analytic number theory (Japanese), Iwanami Shoten 1950.